Base Fee Manipulation In Ethereum's EIP-1559 Transaction Fee Mechanism

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Abstract

In 2021 Ethereum adjusted the transaction pricing mechanism by implementing EIP-1559, which introduces the *base fee* — a fixed network fee per block that is burned and adjusted dynamically in accordance with network demand. The authors of the Ethereum Improvement Proposal (EIP) noted that a miner with more than 50% of the mining power might have an incentive to deviate from the honest mining strategy. Instead, such a miner could propose a series of empty blocks to increase its future rewards. In this paper, we generalize this attack and show that under rational player behavior, deviating from the honest strategy can be profitable for a miner with less than 50% of the mining power. Further, even when miners do not collaborate, it is sometimes rational for smaller miners to join in on the attack.

1 Introduction

Ethereum is a cryptocurrency with a market capitalization of approximately 220 billion US dollars as of the time of writing.[1] In April 2019, the Ethereum Improvement Proposal #1559 (EIP-1559) [4] was proposed and later deployed on Ethereum's mainnet in August 2021 and to this day remains Ethereum's transaction fee mechanism. EIP-1559 seeks to improve the user experience by introducing a new transaction fee mechanism, replacing the previous first price auction mechanism. One of the main goals of EIP-1559 is to simplify the bidding process and reduce the need for complex fee estimation algorithms, while also ensuring that the mechanism is both truthful and does not incentivize miner or user bribes, as articulated by Roughgarden [20]. Broadly speaking, EIP-1559 has been shown to be incentive compatible when miners are assumed to be myopic [20, 21] and it has succeeded in making fees easier to estimate and in reducing delays [17].

EIP-1559 represents a departure from the previous first-price auction system. Prior to EIP-1559, miners were awarded the entirety of the fee paid by users for including their transactions in a block. EIP-1559 introduced a base fee, which is a portion of the transaction fee that is burned and not awarded to miners, and which varies according to the fill rate of blocks. A target block size is defined, such that blocks exceeding the target size increase the base fee and blocks below the target size decrease the base fee.

Miners are compensated for creating blocks through both a block reward and tips from users. However, the dynamic nature of the base fee, which is influenced by the fill rate of blocks, opens the door for manipulation by miners. As they have control over the fill rate of blocks, miners may choose to mine empty blocks in order to decrease the base fee and increase their profits, i.e., tips paid on top of the base fee by users. This genre of strategies has been acknowledged in Ethereum's EIP-1559 proposal and has been explored in previous research works [20, 10], but there is still room for further investigation on this topic.

This paper presents an analysis of the potential for minority (e.g., 20% cf. Section 4) incentive attacks on Ethereum's EIP-1559 transaction fee mechanism. Our results demonstrate that even under the conservative assumption of a steady demand curve, the mechanism is vulnerable to such attacks.[2] Additionally, we show that smaller miners may be incentivized to join in on the attack. We provide general insights into when deviating from the prescribed strategy is rational and note that, due to the nature of our model and assumptions, the results are applicable in a wide range of scenarios. We also explain how the attack can be initiated by an Ethereum user (rather than a miner), i.e., show the

[1] According to `https://coinmarketcap.com/` accessed at 25APR2023.

[2] It is interesting to mention that mining pools with more than 20% power exist both in the Ethereum network `https://beaconcha.in/pools`, and in the Bitcoin network `https://btc.com/stats/pool`.

incentives of users to enact bribes. To address the identified vulnerability, we propose a mitigation and evaluate its effectiveness through simulations.

2 Basic block reward mechanism in Ethereum

Block proposals. Whether Ethereum's blockchain relies on proof-of-work (PoW) or proof-of-stake (PoS) as sybil resistance, it relies on a leader elected at each block to propose new blocks of transactions. Our model covers both PoW and PoS, hence, the results of this paper apply to both types of blockchains. In a PoW blockchain, miners compete to solve a unique computational puzzle and the likelihood of a miner being chosen is based on their share of the network's computational power. In a PoS blockchain, miners stake amounts of the blockchain's native currency to participate and are randomly selected to create a new block with probability proportional to the amount they stake. In both cases, the ideal process is memoryless so each leader election is independent of the previous one.

When a miner is chosen to propose a block, they select a set of unspent transactions from the network to include in the block and broadcast it to the network. Upon successful inclusion of their block in the blockchain, the miner receives two rewards: a fixed reward of newly minted currency (Ether in Ethereum's case), and a variable reward from the transaction fees of the included transactions. The block reward is fixed, regardless of the block's content or the miner that proposes it, so our analysis focuses on the potential for miners to increase their revenue via transaction fees. Therefore, we do not consider the block reward.

Transaction fees under EIP-1559. Ethereum transactions involve a set of instructions that are carried out by miners when the transaction is added to the blockchain. To prevent users from overwhelming the network with bogus transactions, users must pay *transaction fees*. These (fees) should reflect the amount of computational resources needed to execute the instructions, measured in units of *gas* and priced in *Gwei*. (1 Gwei $\triangleq 10^{-9}$ Ether)

An EIP-1559 transaction fee includes a *base fee*, which is paid per unit of gas and varies to balance the supply of gas with the demand for gas. To be exact, the base fee $b[i]$ for block i is determined from the base fee $b[i-1]$ and size $s[i-1]$ of the previous block as follows:

$$b[i] \triangleq b[i-1] \cdot \left(1 + \phi \cdot \frac{s[i-1] - s^*}{s^*}\right). \tag{1}$$

Thus, the base fee is determined by comparing the size (measured in gas) of the previous block to a target block size s^*. (The maximum block size is $2s^*$.) If the block is larger than the target size, it indicates high demand for gas and the base fee is increased to decrease demand. Conversely, if the block size is smaller than the target, it indicates low demand for gas, and the base fee is decreased to increase demand. The sensitivity of the base fee to the size of the previous block is determined by the adjustment parameter ϕ that is currently set to $\frac{1}{8}$ on Ethereum.

When creating a transaction under the EIP-1559 mechanism, in addition to the gas limit g, the user must specify the fee cap c which is the maximum fee per gas unit they are willing to pay, and a maximum tip per gas unit ε which is the priority fee. The transaction will be included in a block only if the fee cap is greater than or equal to the base fee (b). The total fee paid by the user is $\tilde{g} \cdot \min(b+t, c)$, where $\tilde{g} < g$ is the actual gas consumed by the transaction.[3] A portion of the fee $\tilde{g} \cdot b$ is burnt and the remaining $\tilde{g} \cdot \min(\varepsilon, c-b)$ goes to the miner as a tip. The EIP-1559 mechanism aims for users to bid small tips that only cover a miner's costs [4, 20]. Miners are intended to include all transactions that have a fee cap greater or equal to the base fee and prioritize transactions with higher fees only if the maximal block size (s_{max}) is exceeded. For simplicity, we assume that all transactions are of the same size and have sufficient gas limit (i.e., $g = \tilde{g} = 1$) to eliminate considerations including knapsack and gas estimation, and keep the focus on the core matter. (E.g., a large transaction is modeled by multiple smaller transactions.)

3 Model and Assumptions

In the following we present our model and outline our assumptions. We highlight that our model follows Roughgarden's [20] very closely with the exception that we do not restrict miners to be myopic. Note that Roughgarden [20] considers only immediate profits, we on the other hand, will also consider future

[3] The amount of gas needed to execute a transaction is not always predictable in advance. Moreover, if the needed amount of gas exceeds g, then g gas units are consumed and payed for but the transaction fails.

profits.

Users. We assume that users are rational agents who want their submitted transactions to be processed and included in the blockchain. The cost of having one's transaction, tx, included in the blockchain under EIP-1559 depends on the base fee b, fee cap c_{tx}, and the maximum tip t_{tx}, which we described in Section 2. Each transaction tx has a value v_{tx} that is private to the user who proposes it, which can be thought of as the maximum price that the user is willing to pay for the transaction tx to be included in the blockchain. We, therefore, take the utility of a user proposing tx at each block to be $u(tx) = v_{tx} - \min(b + t_{tx}, c_{tx})$ if tx is included and 0 otherwise. A transaction's value is determined by the user, who will then adjust c_{tx} and t_{tx} as part of their bidding strategy, while b is determined by the size of the past blocks according to Equation 1. For ease of exposition, we may sometimes prefer to describe the user's bid for transaction tx with $\langle b_{tx}, t_{tx} \rangle$ instead of $\langle c_{tx}, t_{tx} \rangle$, where the value of $b_{tx} \triangleq c_{tx} - t_{tx}$.

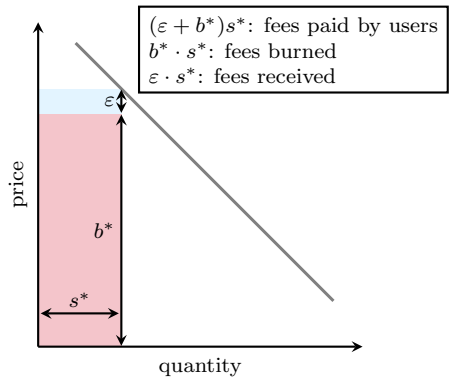
Miners. Miners produce blocks that include transactions to be executed. The miner to propose the next block is chosen at random. Drawings constitute independent experiments, and in each drawing, a miner X is chosen with probability p_x, where p_x corresponds to the miner's share of the total network power. We assume that the power distribution in the network changes slowly, hence, to simplify the analysis we model the system as having a fixed network power distribution, i.e., miners' network power does not change with time.

A miner has dictatorial power over which transactions to include in the block it produces. We assume that miners are rational agents that wish to maximize their profit and therefore choose which transactions to include in their block according to a strategy that maximizes their profit. Aside from their transaction picking strategy, we assume that miners behave "honestly" – that is, as specified by the protocol. We highlight that by not considering other forms of deviating from the protocol, we strengthen our result, showing that even "practically honest" miners would deviate from EIP-1559.

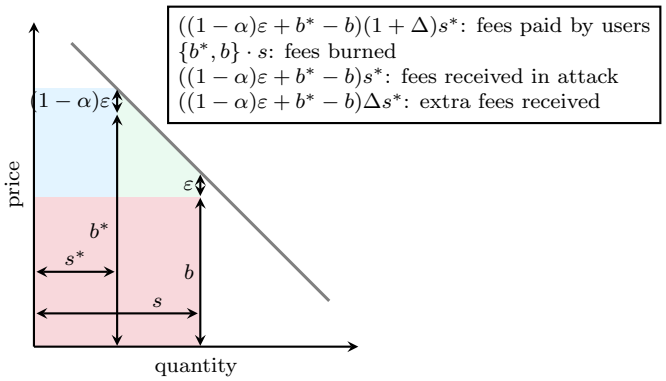
Collusion. We adopt a standard buyers-sellers perspective. We assume that miners do not collude with each other, as they can be viewed as one miner with more power. Similarly, we assume that users do not collude with one another since they are competing for the same scarce resource. However, a user (buyer) and a miner (seller) can communicate and adjust their strategies for mutual benefit. It's worth noting that a miner-user agreement can be reached without much private communication, just by observing each other's actions in real-time.

Steady State. The distribution of prices that transactions are valued at is represented by a demand curve, and the standard demand curve is a decreasing function, meaning that the higher the fee is, the fewer transactions are willing to pay it. (Figure 1a visualizes a sample demand curve.) We consider a system in *steady state*, which we define as a system in which the demand curve does not change over time, i.e., it is the same whenever a miner creates a block. This steady state assumption is good for several reasons: (1) it keeps us on par with the most Roughgarden's work [20], (2) it simplifies the analysis by removing noisy components that can obscure the core principles, and (3) it is a conservative assumption that strengthen our results in comparison to others. For example, the "steady influx" model — where there is a fixed influx of transactions to the mempool and the miners can cause artificially high demand by delaying the inclusion of transactions — assumes that miners are able to manipulate the demand curve in their favor. This, in particular, means that when the attack strategy of Section 4 is beneficial under the *steady state* it will also be beneficial (and perhaps more so) under the "steady influx" model, but not vice versa. Hence, our steady demand curve assumption makes our results more general and robust, as it applies to a wider range of adversarial assumptions.

The desired dynamics of EIP-1559 in the steady state are that blocks remain in the target size s^* and that the base fee remains constant for all blocks, i.e., $s[i] = s^*$ and $b[i] = b^*$ for all i. We refer to these sizes as the target size and the target base fee. Further, EIP-1559's desired bidding dynamics are for the user's optimal strategy to be honest in reporting the value of a transaction via c_{tx} and to offer a minimal tip. In other words, in the steady state, EIP-1559 should lead to a Nash-equilibrium with the following strategies: (users) honest value-reporting, and (miners) including all txs with $c_{tx} < b^*$. Then a block produced during steady state would include s^* transactions that are each paying $b^* + \epsilon$ in fees. Thus, $b^* \cdot s^*$ is burned (red area in Figure 1a), and $\epsilon \cdot s^*$ is received by the miner (blue area in Figure 1a). We note that the desired dynamics are achieved according to the analysis in [20] for immediate-profit-only miners.



(a) Sample steady state demand curve, s^* transactions are willing to pay $b^* + \varepsilon$, where b^* is the base fee that corresponds to the target size. The red area is the amount of fees being burned, whereas the blue area are the fees received by the miner.



(b) Sample steady state demand curve with lowered base fee b. In case the base fee was lowered to $b < b^*$, the demand is increased to $s \geq s^*$. An honest miner will fill up the block with all transactions, paying at least the base fee. The red area indicates the amount of fees being burned, the blue area the fees the attacking miner X receives and the green area are the additional fees received by the honest miner Y for including all transactions.

Figure 1: Example demand curves for Ethereum transactions. The quantity (x-axis) indicates the number of the transactions willing to pay the transaction fee (price shown y-axis).

4 A Miner's Deviation from the Honest Strategy

Consider a miner X who controls a proportion p_x of the network's mining power, i.e., the probability that a miner propose the next block is p_x. The honest strategy for X is to always include the maximum possible number of transactions whose gas fee covers at least the base fee. As we consider a system in steady state, the demand curve does not change over time. Thus, miner X will always propose a block that is exactly the target size s^*. The payout received by miner X for every proposed block is, therefore, $s^* \cdot \varepsilon$, where ε is the tip the miner receives from the users.

In the following, we outline a strategy miner X can employ to manipulate the base fee and increase his profits. When X proposes a block, for which the preceding block was not created by X, he proposes an empty block to reduce the base fee b for subsequent blocks. The miner will receive no payout for this block. Any other consecutive blocks X is chosen to propose, he will propose at size s^* (at target size), profiting from the difference between the targeted and the reduced base fee. As we assume collaboration between the miners and users, as well as consider a system in steady state, we assume in the following that users will continue to submit transactions with total gas price $b^* + (1-\alpha)\varepsilon$, where $0 \leq \alpha \leq 1$. Thus, the miner will receive at least the difference in base fees and the users will pay no more than they would have payed if the miner had not artificially reduced the base fee. More precisely, whenever X proposes a s^* sized block with an artificially reduced base fee, X receives

$$s^* \left(\phi \cdot b^* + (1-\alpha)\varepsilon\right), \tag{2}$$

where $\phi \cdot b^*$ is the base fee reduction and α represents the proportional reduction of the tip paid by the users. For simplicity, we assume the attack finishes whenever miner X's consecutive turns as proposer finishes.[4]

In Theorem 1 we compute the expected reward of X following the honest strategy, as well as the aforementioned described deviation from the honest strategy. By comparing the payout of consecutive turns of X in both strategies, we find that it is rational also for a miner with less than 50% of the power to deviate from the honest strategy.

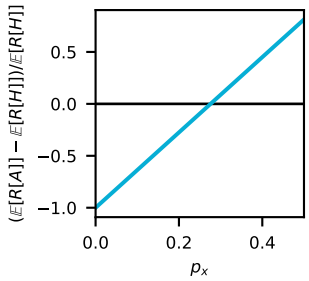
Theorem 1. *In expectation, it is rational for miner X to deviate from the honest strategy, if*

$$p_x > \frac{\varepsilon}{\phi \cdot b^* + (1-\alpha)\varepsilon}.$$

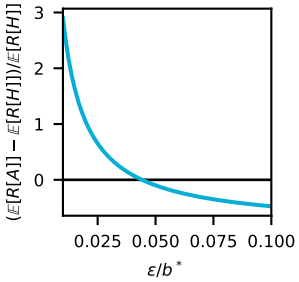
Proof. See Appendix A. □

To better illustrate, when it is a rational behavior for miner X to deviate from the honest strategy, we plot the relative difference between the expected reward of the attack and the honest strategy in

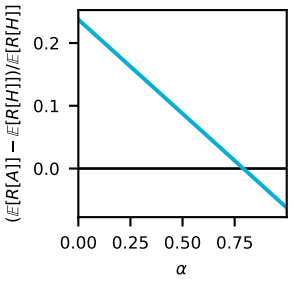
[4]The simplification does not affect the generality of our results. In fact, the deviation might only become more profitable if we allow X to continue the attack after an honest proposer, hence, our results apply without the simplification as well.



(a) Relative difference shown as a function of p_x. We set $\alpha = 0.5$, $\varepsilon/b^* = 1/25$.



(b) Relative difference shown as a function of ε/b^*. We set $p_x = 0.3$, $\alpha = 0.5$.



(c) Relative difference shown as a function of α. We set $p_x = 0.3$, $\varepsilon/b^* = 25$.

Figure 2: Relative difference between expected reward of attack and honest strategy as a function of p_x (cf. Figure 2a), ε/b^* (cf. Figure 2b) and α (cf. Figure 2c). We set $\phi = 1/8$, as set in Ethereum, in all plots. It is rational for X to perform the attack whenever the relative difference is positive.

Figure 2. For many realistic parameter configurations it is rational behavior for miner X to perform the attack and thereby manipulate the base fee. In Figure 2a, we plot the profitability of the attack in comparison to the honest strategy dependent on X's mining power p_x. We set $\phi = 1/8, \varepsilon/b^* = 1/25$, and $\alpha = 0.5$. Notice that even miners with a mining power less than 0.3 are expected to profit from executing the attack. To underline the expected profitability of the attack, even for small miners, we vary the ratio between the tips and the base fee (ε/b^*) in Figure 2b and set $p_x = 0.3$. Additionally, we vary α, the share of the tips the users keep for themselves, in Figure 2c, and again set $p_x = 0.3$. We conclude that there are multiple imaginable and realistic parameter configurations for which the outlined attack is profitable, even for a miner with less than 50% of the mining power. Thus, as individual Ethereum pools control in excess of 20% of the mining (staking) power such an a attack is realistic.[5]

5 The Attack's Effect on Other Miners

In the following, we consider a scenario where a miner X with staking power p_x ($0 < p_x < 1$) exists for which it is rational behavior to perform the base fee manipulation studied in Section 4. We then analyze the effect of a miner Y with staking power p_y, where $0 < p_y < p_x$, observing that X performs the base fee manipulation attack. More precisely, we study whether it is rational behavior for Y to join the attack partially, i.e., Y will always propose blocks at target size and thereby help keep the base fee artificially low in Section 5.1. Additionally, we will also study when Y would rationally join the attack entirely, i.e., Y also proposes empty blocks when the base fee is not already artificially lowered in Section 5.2. We remark that, throughout, we always assume that miners X and Y do not collaborate.

5.1 Joining the Attack

Now consider a miner Y that observes X performing the attack outlined from Section 4. Miner Y is selected as the proposer for a block that follows X's turn as proposer, i.e., the base fee is currently artificially lowered to $(1-\phi)b^*$. We analyze whether it is rational for Y to follow the honest strategy, i.e., propose the largest block possible and thereby increase the base fee again, or to join the attack and continue keeping the base fee artificially low.

We first describe the honest strategy. When it is miner Y's turn to propose a block at an artificially lower base fee, miner Y proposes a block with the maximum number of transactions possible. The number of transactions is restricted both by the demand at the current base fee b, as well as the maximum block size, which is twice the target size ($2s^*$). Consider the demand curve shown in Figure 1b, the demand at price $b^* + \varepsilon$, where b^* is the target base fee price and ε the tip, corresponds to a block of target size s^*. Miner Y will propose a block with the artificially lowered base fee $b = (1-\phi)b^*$. The demand at this new price $b + \varepsilon$ is represented as s in Figure 1b. We make no assumptions about the shape of the demand curve. But to make our results stronger, we consider the best case for the honest strategy. Namely, due to the increased demand, Y can propose a block of maximum size (and reap the resulting extra rewards). That is, $s = 2s^*$, after X's turn. Thus, the payout for miner Y proposing a block of size $2s^*$ is given by

$$s^* \left(\phi \cdot b^* + (1-\alpha)\varepsilon\right)(1+\Delta), \tag{3}$$

[5] https://beaconcha.in/pools

where $s^*\left(\phi \cdot b^* + (1-\alpha)\varepsilon\right)$ is the payout the attacking miner X would receive (cf. Equation 2) and $\Delta \in [0,1]$ is a scaling factor that dictates how much additional rewards miner Y received for mining a maximum size block. While $\Delta = 0$ would indicate that Y earns exactly as much as X, i.e., all extra transactions are capped at exactly the base fee, $\Delta = 1$ would indicate that miner Y earns twice the rewards of X, i.e., all extra transactions are capped at the highest possible price of $b^* + \varepsilon$.

After miner Y proposes the block, he will be chosen to also propose the following block with a probability of p_y. We continue with the approximation from before, i.e., the effect of a full block after an empty one leads approximately to the same point on the demand curve – $(s^*, b^* + \varepsilon)$. [6] Thus, miner Y proposes a block of size s^* and is awarded $s^* \cdot \varepsilon$ for proposing the block and, from thereon out, will continue doing so until its consecutive turns as a proposer finishes.

With a probability of p_x, miner X will interrupt Y's turn as proposer, and we assume that miner X will start the attack again and propose an empty block to lower the base fee. Note that our approximation of the base fee returning to steady-state levels is the best case for Y's honest strategy and, thus, makes our results stronger. For all consecutive blocks proposed by X, miner X will propose target size blocks to keep the base fee b (i.e., $(1-\phi)b^*$). If miner Y is again selected as a proposer after X's turn, miner Y will proceed according to the previously outlined honest strategy.

At any point, with a probability of $1 - p_y - p_x$, the consecutive turn of the two miners finishes. Note that for Y's honest strategy analysis, we are only interested in these consecutive turns of the two miners as proposers, as we analyze the expected payout of the same sequences for the deviation from the honest strategy which we outline in the following.

We now describe a strategy Y can employ to join the attack it observes. When it is Y's turn to propose a block and the base fee was artificially lowered by X, miner Y will propose a block at target size. Equivalently to the payout received by miner X for such a block (cf. Section 4), miner Y's reward for proposing the block is given by

$$s^*\left(\phi \cdot b^* + (1-\alpha)\varepsilon\right). \tag{4}$$

Following the block proposed by Y, miner Y is again selected to propose a block with probability p_y and miner X with probability p_x. As long as the two miners have an uninterrupted sequence of block proposals, they both keep the base fee artificially low by always proposing target size blocks. Once their consecutive turn as proposers ends, we consider the attack finished. In Theorem 2, we show that it can be profitable for such a miner Y with a mining power p_y to join the attack, i.e., keep the base fee artificially low, if it see a miner X with a mining power $p_x > p_y$ perform the attack. Note that this is without assuming collaboration between the two miners.

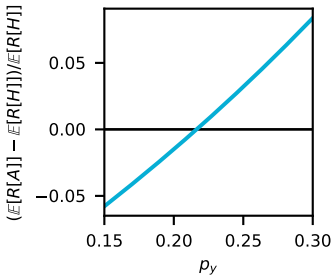
Theorem 2. *In expectation, it is rational for a miner Y to deviate from the honest strategy and join X in keeping the base fee low, if*

$$p_y > \frac{\Delta((1-\alpha)\varepsilon + \phi \cdot b^*)}{(1-\alpha)\Delta \cdot \varepsilon + (1+\Delta)\phi \cdot b^* - \alpha \cdot \varepsilon}.$$

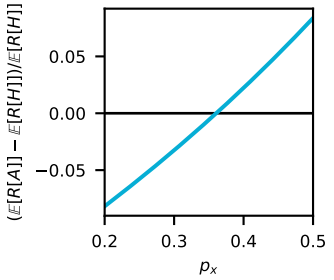
Proof. See Appendix A. □

To better gauge when it is profitable for miner Y to join the attack, we plot the relative difference between the expected return from the outlined attack and the expected return from the honest strategy in Figure 3. We vary the mining powers of miner Y (cf. Figure 3a) and miner X (cf. Figure 3b), as well as Δ (cf. Figure 3c). In all three plots we set $\phi = 1/8, \varepsilon/b^* = 1/25$, and $\alpha = 0.5$. In Figure 3a, we set $\Delta = 0.2$ and notice that even a miner Y with a mining power slightly larger than 0.2 would be inclined to join the attacking miner X with a mining power of 0.3 in manipulating the base fee. We observe in Figure 3b that a miner Y that is only six-tenths of the size of miner X would also join the attack even if miner X only controls less than 40% of the mining power. Finally, in Figure 3c, we show the dependency of the attacks profitability for miner Y as a function of Δ, i.e., the additional payout received by miner Y following the honest strategy when proposing a full block after the attack by X. Notice that while it is rational for a miner Y to join the attack for small Δ's, this is not the case for larger Δ's. We remark that this is due to the rewards from the full block mined if Y follows the honest strategy being very significant for a large Δ. Nevertheless, our results show that for realistic parameter configurations, it is rational for a miner Y to join the attack it sees a larger miner X perform — even without assuming collaboration between the two.

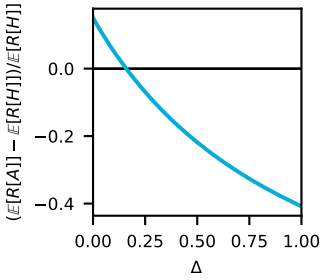
[6] In addition to the reason given in Section 4, we note that this approximation is accurate up to a term in $O(\phi^2 b^*)$ while $b \in \Theta(\phi b^*)$.



(a) Relative difference shown as a function of p_y. We set $p_x = 0.3$, $\Delta = 0.2$.



(b) Relative difference shown as a function of p_x. We set $p_y = 0.6p_x$, $\Delta = 0.2$.



(c) Relative difference shown as a function of Δ. We set $p_x = 0.3$, $p_y = 0.18$.

Figure 3: Relative difference between expected reward of attack and honest strategy as a function of p_y (cf. Figure 3a), p_x (cf. Figure 3b) and Δ (cf. Figure 3c). We set $\phi = 1/8$, as implemented in Ethereum, $\varepsilon/b^* = 1/25$, $\alpha = 0.5$ in all plots. It is rational for Y to join the attack whenever the relative difference is positive.

5.2 Join and Initiate the Attack

In addition to only joining the attack, it is also possible for miner Y, observing X continuously performing the base fee manipulation, to also propose an empty block whenever it proposes a block with the target base fee (b^*), knowing that X will aid it in keeping the base fee low subsequently. We analyze, in the following, when it is more profitable for miner Y to join X's attack in its entirety, as opposed to remaining honest.

The honest strategy for miner Y corresponds to the honest strategy described in Section 5.1. However, now it is also important to mention that anytime miner Y proposes a block with base fee b^*, i.e., whenever Y proposes a block that does not follow X's attack, Y will propose a target size (s^*) block. For proposing such a block, miner Y will receive $s^* \cdot \varepsilon$.

In the deviation from the honest strategy, on the other hand, Y will propose an empty block whenever it mines a block where the base fee corresponds to the target base fee b^*. Then with probability p_y, miner Y will also propose the next block and will profit from the difference between the base fee and the target base fee by mining a target size block. On the other hand, with probability p_x, miner X will propose the next block and will also mine a target size block – keeping the base fee low.

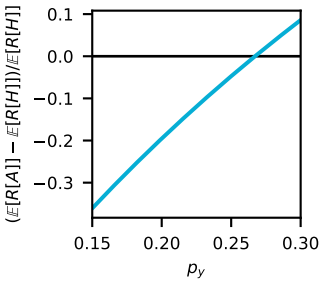
With Theorem 3, we show that it can even be profitable for a miner Y to commence the attack if it knows that a larger miner X will help it in keeping the base fee artificially low.

Theorem 3. *In expectation, it is rational for a miner Y to deviate from the honest strategy and lower the base fee, if*

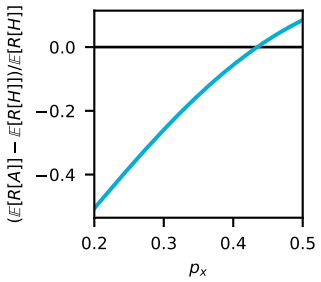
$$p_y > \frac{\varepsilon(1-p_x)}{(1-\alpha)\varepsilon + \phi \cdot b^*(1-p_x) + (1+\Delta)\varepsilon\alpha p_x - \Delta p_x(\varepsilon + \phi \cdot b^*)}.$$

Proof. See Appendix A. □

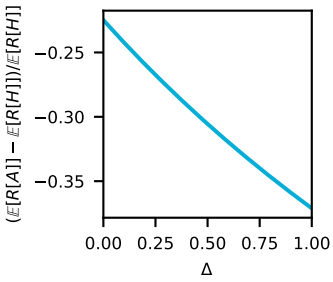
To better understand the parameter configuration under which it would be rational for Y to also initiate the attack, we plot the relative difference of the reward of the attack in comparison to the reward of the honest strategy in Figure 4. Note that we again set $\phi = 1/8, \varepsilon/b^* = 1/25$, and $\alpha = 0.5$. Figure 4a, where we also set $p_x = 0.3$ and $\Delta = 0.2$, shows that it can be profitable for a miner Y to initiate the attack, i.e., mine an empty block to lower the base fee, knowing that miner X will support it in keeping the base fee artificially low. Notice though that the threshold where it is rational for Y to also start the attack is reached later in comparison to the threshold where it is rational for miner Y to only join the attack (cf. Figure 3a). A similar picture paints itself when we look at the profitability of initiating the attack for Y as a function of X's mining power in Figure 4a. Again we see that for a miner Y, it can be profitable to start the attack only with the knowledge that X will aid it in keeping the base fee low. Finally, in Figure 4c, we show the relative difference between the expected profit of the outlined attack for Y and the honest strategy. For the chosen parameter configuration, $p_x = 0.3$ and $p_y = 0.18$, the attack would actually never be profitable regardless of Δ. We, thus, summarize that while it is only rational for Y to initiate the attack for a reduced set of parameters, it is remarkable that this is even the case. If Y also starts the attack, it is taking a loss for a larger miner X just based



(a) Relative difference shown as a function of p_y. We set $p_x = 0.3$, $\Delta = 0.2$.



(b) Relative difference shown as a function of p_x. We set $p_y = 0.6p_x$, $\Delta = 0.2$.



(c) Relative difference shown as a function of Δ. We set $p_x = 0.3$, , $p_y = 0.18$.

Figure 4: Relative difference between expected reward of attack and honest strategy as a function of p_y (cf. Figure 3a), p_x (cf. Figure 3b) and Δ (cf. Figure 3c). We set $\phi = 1/8$, as implemented in Ethereum, $\varepsilon/b^* = 1/25$, $\alpha = 0.5$ in all plots. It is rational for Y to initiate new attacks in addition to X whenever the relative difference is positive.

on the knowledge that it will be supported by X in keeping the base fee low but without assuming any collaboration between the two.

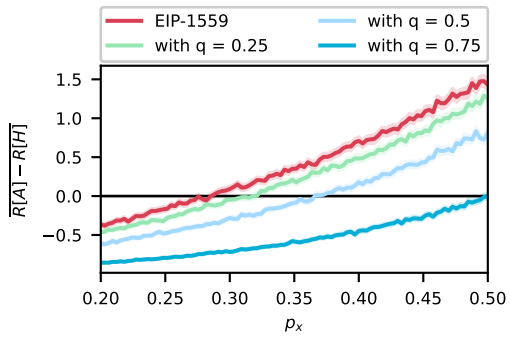
6 Possible Mitigations

To mitigate the attacks described in Sections 4 and 5, we focus on addressing the deviation of player X, as if X does not deviate from the honest strategy, neither does Y. We start by examining the trivial mitigation of reducing ϕ. Theorem 1 shows that decreasing ϕ by a factor of β, will require that p_x will be approximately β times larger for a deviation to be profitable. For example, if Ethereum were to decrease its current ϕ of 1/8 to 1/16, it would require p_x to be approximately twice as large for a deviation to be profitable. This approach can be effective during stable periods, but it might not be able to adjust quickly enough to changes in demand. Further, Leonardos et al. [15] show how the value of ϕ determines the trade-off between a base fee that adjusts too quickly (which can lead to chaotic behavior) and one that adjusts too slowly to fulfil its purpose.

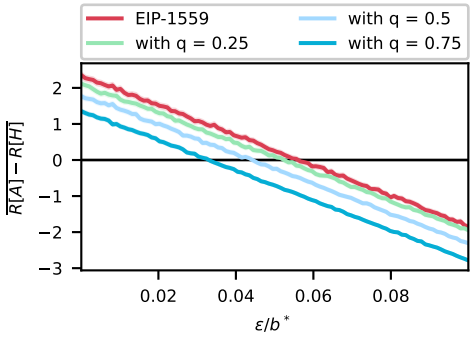
The following question appears in EIP1559 FAQ [3]: *"Won't miners have the incentive to collude to push down the base-fee by making all their blocks less than half full?"* In response to the question Buterin proposes this mitigation: Divert half of the collected base fees, that would otherwise be burned, into a special pool. Whenever a miner mines a new block add to its block reward a 1/8192 portion of the amount in that pool. This will incentivize miners to maintain a higher base fee. One might falsely presume that this solution, to the above posed question, might also be used to solve the deviation we outline in Section 4 despite the fact that the two attacks are inherently different (the one we outline does not require miners to collude). This proposal does not solve our deviation since the added cost of the attack, i.e., the lost revenue from half of the base fee reduction, is distributed among many while the attacker's expected revenue remains unchanged. In the long run, this proposal only minimally reduces the attacker's expected revenue by $b^* \cdot 2^{-13}$, which is typically significantly less than the expected rewards and, therefore, ineffective.

Another straw-man to consider is using the average of the previous W block sizes instead of just the size of the last block. This method may appear to be promising because it reduces the effect of an empty block on the following block by a factor of W, and its effect on the rate of adjustment (embodied in ϕ) is easily accounted for, adding an adjustment delay within $O(W)$. However, it is not a good solution; it simply fails to mitigate the attack. The W-window approach increases the opportunity for X to profit from later blocks that are within a W distance from the empty block, thereby increasing the expected profitability of the deviation. For example, if we use a two-block window (i.e., $W = 2$) and $\phi = 1/8$, the base fee is reduced by a smaller factor of only $\phi/W = -1/16$ to $b_1 = 15b^*/16$ as desired. Nevertheless, even if the ensuing block is completely full (i.e., $s_1 = 2s^*$), the base fee for the block following it does not increase and remains $b_2 = b_1 = 15b^*/16$ which means an additional profit opportunity for the deviation that compensates X for the reduced factor. As a result, the deviation is not mitigated and is actually exacerbated.

We propose the following mitigation, to use a geometric sequence as weights to average the history



(a) Difference of mean attack and mean honest return as a function of p_x for EIP-1559 and the proposed mitigation for $q \in \{1/4, 1/2, 3/4\}$. We set $\varepsilon/b^* = 1/25$.



(b) Difference of mean attack and mean honest return as a function of ε/b^* for EIP-1559 and the proposed mitigation for $q \in \{1/4, 1/2, 3/4\}$. We set $p_x = 0.4$.

Figure 5: Profitability of the attack under EIP-1559 and our proposed mitigation. We plot the mean attack profitability (cf. Figures 5a and 5b) along with the 95% confidence interval. We set $\phi = 1/8$, $\alpha = 0.5$, $s^* = 1$.

of block sizes. Formally, for $q \in (0, 1)$ denote

$$s_{avg}[i] \triangleq \frac{1-q}{q} \sum_{k=1}^{\infty} q^k \cdot s[i-k+1] = (1-q) \cdot s[i] + q \cdot s_{avg}[i-1], \tag{5}$$

and replace $s[i]$ in Equation 1 by $s_{avg}[i]$ to get the following base-fee update rule

$$b[i+1] = b[i] \cdot \left(1 + \phi \cdot \frac{s_{avg}[i] - s^*}{s^*}\right). \tag{6}$$

By applying the update rule in Equation 6, we reduce the effect of the empty block on the ensuing block by a factor of $(1-q)$, and discount its effects on later blocks exponentially fast (with base q). For example, using the same parameters as before with $\phi = 1/8$, if we set $q = 1/2$, after X's empty block (at slot τ), the base fee will be reduced by a factor of $(1 - \phi(1-q)) = (1 - 1/16)$ to $b[\tau+1] = 15b^*/16$. Now, however, making the same assumption as before (i.e., $s[\tau+1] = 2s^*$), the base fee for the next block, $b[\tau+2]$, will be $495b^*/512$. This decreases the potential profit margin of block $\tau+2$ from $b^*/16$ to $b^*/32 + b^*/512$, which is almost a factor of 2 reduction. Therefore, the added profit opportunity in future blocks is not enough to compensate for the lost potential in the immediately ensuing block. As a consequence, the attack is considerably mitigated.

The above mitigation method has two additional properties: (i) its computation and space complexity are both in $O(1)$, and (ii) it gradually phases out the impact of a single empty block without causing significant fluctuations. To reason about the effect our proposal has on response times we use the following methodology. Suppose that the demand suddenly changes and the new (desired) steady state should be reached at a new point with a base fee that is β times higher than the current base fee (b^*). Denote by T the number of consecutive full blocks required to reach the new base fee. We use T as a function of β to characterize the response time of a fee-setting mechanism to sudden changes in demand.

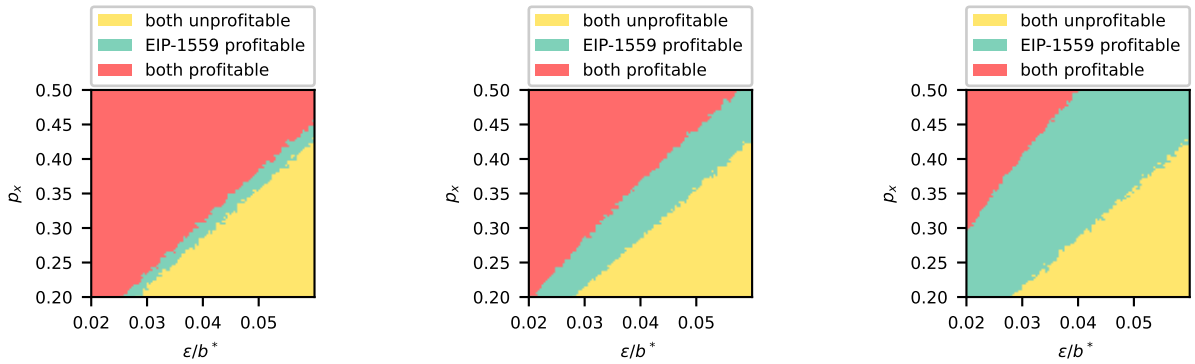
EIP-1559 as it is currently implemented will take $T(\beta) = \lceil \log_{(1+\phi)} \beta \rceil$ blocks to reach the new base fee βb^*. With the mitigation method it takes slightly longer. To be precise, in Appendix B we show that with our mitigation proposal the exact delay $T(\beta)$ is the smallest integer that satisfies

$$\prod_{k=1}^{T} (1 + \phi(1 - q^k)) \geq \beta. \tag{7}$$

We further plot the $T(\beta)$ for EIP-1559 and our mitigation in Appendix B (cf. Figure 10).

6.1 Simulations

In order to gauge the effectiveness of the proposed mitigation, we conducted simulations that compare the excess profit of an attacker (profit gained through deviation minus profit gained through honest mining) under EIP-1559 with and without the mitigation. To account for the probabilistic nature of the attack (profits in expectation only), we calculated the average of the results for each data point over 10'000 runs, each using a different random seed. Each simulation begins with X mining an empty block, the next blocks are mined by X with probability p_x per block and by an honest miner with



(a) $q = 1/4$ (b) $q = 1/2$ (c) $q = 3/4$

Figure 6: Attack profitability under EIP-1559 and the proposed mitigation for $q \in \{1/4, 1/2, 3/4\}$ as a function of p_x and ε/b^*. Importantly, the green area shows where the mitigation can prevent the attack but EIP-1559 cannot. We set $\phi = 1/8$, $\alpha = 0.5$, $s^* = 1$.

probability $1 - p_x$. We assume that X will then always mine target size blocks, while the honest miners will mine blocks at twice the target size. We keep track of the payout received by X and declare the attack finished if the base fee has recovered to 99% of its target size. We perform 10'000 runs for both the base fee evolution according to EIP-1559, as well as our mitigation. Simultaneously, we keep track of the payout X would have received for the same random seed if it had followed the honest strategy.[7] The results are depicted in Figures 5 and 6.

Figures 5a and 5b show X's profit from attacking as a function of p_x and the ratio ε/b^* for EIP-1559 with and without the mitigation for $q \in \{1/4, 1/2, 3/4\}$. The results decisively demonstrate the benefit of the mitigation method, i.e., it which makes it much harder for X to profit from attacking. Finally, Figure 6 illustrates the effect of the mitigation (with $q \in \{1/4, 1/2, 3/4\}$) on the configurations at which X will attack. Most notably, the green area represents a set of configurations in which X had attacked without the mitigation and will be attacking no longer. The value of the proposed approach is evident from the results.

7 User perspective

Until now, we have approached the topic from the miners' point of view. Considering the users as first-class citizens, and observing the attack through their eyes, contributes a new perspective on the results.

Instead of the miners taking it upon themselves to initiate the attack, we can imagine users who wish to pay lower costs coordinating the attack. Let u be a user (or group of users) that has transactions with a g amount of gas. Assume the other users naively follow the strategy of the desired equilibrium. That is, they bid an honest valuation with an ε tip. The attacker's strategy is as follows. u bribes the miner of the current block (no matter the miner's power) to propose an empty one instead. Any bribe larger than $s^*\varepsilon$ will suffice. Consequently, the base fee reduces in the next block. If the other users naively continue to bid with an ε tip (or they are simply slow to react), u can guarantee the inclusion of its transactions with any tip larger than ε — making the attack profitable whenever $g\phi b^* > s^*\varepsilon$.

8 Discussion

Myopic vs. non-myopic. Roughgarden's analysis [20] of the EIP-1559 protocol suggests that, in the steady state, the honest strategies of the users and miners are incentive compatible. Our opposing conclusion stems from a single different assumption, namely, [20] considers miners that only care for immediate profits (referred to as myopic), while we consider non-myopic miners which do not disregard future profits.

There are valid reasons to model miners as myopic. For example, the proposing turns of a very small miner are sporadic, and accounting for rare future profits is negligible in comparison to the prize at hand. However, there are strong reasons for modeling miners as non-myopic. Measurement studies have shown that the distribution of mining power follows a power law rather than a uniform distribution [8], i.e., the biggest miners control significant portions of the mining power. We note that this has not changed with

[7] We will add the code for reproducing the simulations in the public version of the paper.

Ethereum's switch to PoS [12]. Moreover, the "shorter vision" of smaller miners is accounted for under our model. Our quantitative results show that as a miner's size decreases, the lesser the profitability of the deviations. Finally, from a conceptual perspective, miners are typically players of high stakes (the minimum stake in Ethereum is 32 ETH which is over $50'000 at the time of this writing), and participation also requires some expertise, planning, and locking of assets. Therefore, it does not seem appropriate to consider these players as ones that neglect future considerations.

Additional observations. The deviations described in this paper have an interesting property; they appear to have a win-win-win outcome. The attacker profits, the non-attacking miners profit, and also the users profit by paying a cheaper total gas fee. But not all is rosy; there is a hidden cost involved. In order for users to benefit, they must diligently follow the miners' actions and compute the appropriate response. This increase in complexity for the users is in opposition to one of the main goals of EIP-1559 — simplifying the bidding mechanism and eliminating the need for complex fee estimation algorithms. As a result, sophisticated users take precedence and push naïve users to the back of the line.

It is also worth discussing what happens in reality. Although it is desirable that the leader election process is unpredictable, in practice, this is not the case. Currently, in Ethereum, implementation considerations led to miners knowing their own proposing slots 32–62 blocks in advance [6]. This predictability clearly favors the attacker, which no longer needs to lose tips for the probability of winning more. Instead, the miner only attacks when it is guaranteed to mine at least two blocks in a row. The predictability issue has risen with the move to PoS and was not the case with PoW Ethereum.[8]

9 Related work

Blockchain transaction fee mechanism. Huberman et al. [11] provide an early analysis of Bitcoin's first-price auction. An in-depth exploration of miner manipulation in transaction fee mechanisms was first explored by Lavi et al. [14]. While these works study the first-price auction used is Bitcoin and originally in Ethereum as well, our work studies the incentives for miners to deviate from the EIP-1559 protocol currently used in Ethereum.

Stability of the base fee. As Leonardos et al. [15] show in their theoretical analysis, a key parameter in the base fee mechanism is the adjustment parameter ϕ. In particular, they show that stability is not guaranteed depending on system conditions (e.g., a congested network), especially if ϕ is set to too high a value. However, there are also conditions under which the base fee may have bounded oscillations or even converge, providing stability. In another work, Leonardos et al. [16] show that despite the short-term chaotic behavior on the base fee, the long-term average block size is close to the target size.

Reijsbergen et al. [19] empirically show that a stable base fee may not tell the whole story, however, as even in cases where the base fee remains relatively stable (e.g., between 25 and 35 Gwei) and the block size is on average the target block size, block sizes can fluctuate wildly (as explained by Leonardos et al. [15]), impacting miner revenue. One reason for this is that the currently used value for the adjustment parameter ($\phi = \frac{1}{8}$) is too low during periods where the demand rises sharply (i.e., the base fee does not increase quickly enough) but also too high when demand is stable (inducing fluctuations in block size). [19] suggests, therefore, to make ϕ variable based on the demand. Their work does not consider bribes and is complementary to ours. Their suggested mitigation to the stability issue is to have ϕ adaptive according to an Additive Increase Multiplicative Decrease. Since we do not vary ϕ (but instead update the base fee update rule), an interesting experiment would be to combine our mitigation technique (averaging $s[i]$ geometrically) with their AIMD ϕ setting, and examine the results on data from the real world.

Ferreira et al. [7] show that the first-price auction utilized under EIP-1559 is incentive-aligned for miners, it provides a bad user experience. In particular, they observe bounded oscillation of the base fee in experiments when bidder's all associate the same value with their transaction. While their work studies the stability of the base fee with myopic miners, we study the manipulability of the base fee in the presence of non-myopic miners.

Manipulability of the base fee. Manipulation of the base fee has been a concern since EIP-1559 was proposed [2] as it is straightforward to notice that the base fee could be manipulated downwards

[8] Since the randomness in PoW does not depend on peer-to-peer communication source, it does not have a predictability concern. The predictability issue is a result of implementation constraints for the cryptographic protocols of Verifiable Random Functions (VRFs). Therefore, these issues also appear in other Blockchain systems that rely on VRFs (such as Filecoin Proof of Storage based system [13] which, a fun fact, was the first blockchain to deploy EIP-1559).

by a miner that intentionally mines empty blocks.

The EIP as listed on Ethereum's Github repository acknowledges the possibility of miners mining empty blocks but determines that such a deviation from an honest mining strategy would not lead to a stable equilibrium as other miners would benefit from this (i.e., benefiting from the reduced base fee without the opportunity cost of mining an empty block) [4]. Their belief was therefore that executing such a strategy would require a miner to control more than half the hashing power (the document precedes Ethereum's switch to PoS).

In his exposition of EIP-1559 [20], Roughgarden considered this in the case of a 100% miner (or any miner with greater than 50% of the mining power) that would drive the base fee down to 0 by mining empty blocks then, in order to maximize their revenue, switching to either mining target size blocks in perpetuity, maintaining the base fee at 0 and essentially reverting back to a first price auction, or mining sequences of underfull and overfull blocks (relative to the target size) according to the demand. Roughgarden restricts himself to this case as he does not otherwise consider non-myopic miners, assuming instead that with a high enough level of decentralization, the probability of any miner being elected to propose a block two (or more) times in a row was too low for any miner to consider strategies over multiple blocks.

Similar to us, a concurrent work by Hougaard and Pourpouneh [10] also relaxes the myopic assumption and proves that non-myopic miners would be incentivized to deviate even if they are a minority. However, their analysis relies on several assumptions, that while legitimate, provide the attacker with advantageous conditions in comparison to the more conservative assumptions made in this paper. In particular, [10] relies on miners knowing the exact parameters of the demand distribution of the users (which itself is restricted to each user drawing from a uniform distribution), as well as on miners being able to manipulate the demand curve in favor of the future blocks; inducing artificial future congestion by not including transactions in the current block and letting them accumulate. We do not assume any specific demand curve, only a steady demand curve (described in Section 3), and do not rely on miners inducing congestion (which will, of course, make our attack more profitable), although we do assume that users act rationally and will adapt their strategy if benefits them, while [10] assumes users are passive and do not adapt their strategies in a rational manner.

MEV. Miner/Maximal Extractable Value (MEV) has gained significant attention in the blockchain research community in recent years (e.g., [5, 22, 18]). Similarly to this work, MEV strategies enable a miner to accrue excess profits in comparison to naïve mining. However, in MEV the value comes from analyzing the actual data in the transactions, whereas, in this work, the miner's excess profit comes from manipulating the EIP-1559 mechanism. Therefore, many of the suggested mitigations against MEV [1, 9] (such as obscuring transaction data until inclusion) will not work against our attack.

10 Conclusion and Future Work

The analysis presented in this paper shows that even under very conservative assumptions (steady state, miners cannot induce congestion, unknown demand functions), there are strong incentives for minority miners to deviate from the EIP-1559 protocol. Furthermore, Theorems 2 and 3 show that once an attack begins, previously honest miners' rational response may be to join the deviation and even sometimes initiate new attacks, thus worsening the problem rather than improving it.

To mitigate the problem, we suggested using a weighted average with the weights being a geometric series. This direction seems promising as it has several desirable properties and trade-offs (e.g., balancing attack mitigation with low additional response delays). However, further research rigorously analyzing it in a broader context is warranted.

Many questions remain open regarding EIP-1559. To name but a few: While the suggested deviation captures the essence of the problem, devising an optimal deviation strategy is left as an open problem. What is the right abstraction to evaluate the benefits and shortcomings of EIP-1559 in comparison to a first-price auction? How does it differ from PoW to PoS? On a broader level, questions concerning adaptations of auction mechanisms for a blockchain use case have the potential to be both intellectually challenging and practically important and will hopefully receive more attention from the academic community.

References

[1] Carsten Baum, James Hsin-yu Chiang, Bernardo David, Tore Kasper Frederiksen, and Lorenzo Gentile. Sok: Mitigation of front-running in decentralized finance. *Cryptology ePrint Archive*, 2021.

[2] Tim Beiko. EIP-1559 Community Outreach Report, 2020. https://medium.com/ethereum-cat-herders/eip-1559-community-outreach-report-aa18be0666b5.

[3] Vitalik Buterin. EIP 1559 FAQ, 2021. https://notes.ethereum.org/@vbuterin/eip-1559-faq#Won%E2%80%99t-miners-have-the-incentive-to-collude-to-push-down-the-BASEFEE-by-making-all-their-blocks-less-than-half-full.

[4] Vitalik Buterin, Eric Conner, Rick Dudley, Matthew Slipper, Ian Norden, and Abdelhamid Bakhta. Fee market change for ETH 1.0 chain, 2019. https://github.com/ethereum/EIPs/blob/master/EIPS/eip-1559.md.

[5] Philip Daian, Steven Goldfeder, Tyler Kell, Yunqi Li, Xueyuan Zhao, Iddo Bentov, Lorenz Breidenbach, and Ari Juels. Flash boys 2.0: Frontrunning in decentralized exchanges, miner extractable value, and consensus instability. In *2020 IEEE Symposium on Security and Privacy (SP)*, pages 910–927. IEEE, 2020.

[6] Ethereum. Phase 0 – Honest Validator, 2022. https://github.com/ethereum/consensus-specs/blob/dev/specs/phase0/validator.md#lookahead.

[7] Matheus VX Ferreira, Daniel J Moroz, David C Parkes, and Mitchell Stern. Dynamic posted-price mechanisms for the blockchain transaction-fee market. In *Proceedings of the 3rd ACM conference on Advances in Financial Technologies*, pages 86–99, 2021.

[8] Adem Efe Gencer, Soumya Basu, Ittay Eyal, Robbert Van Renesse, and Emin Gün Sirer. Decentralization in bitcoin and ethereum networks. In *Financial Cryptography and Data Security: 22nd International Conference, FC 2018, Nieuwpoort, Curaçao, February 26–March 2, 2018, Revised Selected Papers 22*, pages 439–457. Springer, 2018.

[9] Lioba Heimbach and Roger Wattenhofer. Sok: Preventing transaction reordering manipulations in decentralized finance. *arXiv preprint arXiv:2203.11520*, 2022.

[10] Jens Leth Hougaard and Mohsen Pourpouneh. Farsighted miners under transaction fee mechanism eip1559. Technical report, IFRO Working Paper, 2022.

[11] Gur Huberman, Jacob D Leshno, and Ciamac Moallemi. Monopoly without a monopolist: An economic analysis of the bitcoin payment system. *The Review of Economic Studies*, 88(6):3011–3040, 2021.

[12] Gareth Jenkinson. 64% of staked ETH controlled by 5 entities — Nansen , 2022. https://cointelegraph.com/news/64-of-staked-eth-controlled-by-five-entities-nansen.

[13] Protocol Labs. Filecoin: A Decentralized Storage Network, 2017. https://filecoin.io/filecoin.pdf.

[14] Ron Lavi, Or Sattath, and Aviv Zohar. Redesigning bitcoin's fee market. *ACM Transactions on Economics and Computation*, 10(1):1–31, 2022.

[15] Stefanos Leonardos, Barnabé Monnot, Daniël Reijsbergen, Efstratios Skoulakis, and Georgios Piliouras. Dynamical analysis of the eip-1559 ethereum fee market. In *Proceedings of the 3rd ACM Conference on Advances in Financial Technologies*, AFT '21, page 114–126, New York, NY, USA, 2021. Association for Computing Machinery. doi:10.1145/3479722.3480993.

[16] Stefanos Leonardos, Daniël Reijsbergen, Daniël Reijsbergen, Barnabé Monnot, and Georgios Piliouras. Optimality despite chaos in fee markets. *arXiv preprint arXiv:2212.07175*, 2022.

[17] Yulin Liu, Yuxuan Lu, Kartik Nayak, Fan Zhang, Luyao Zhang, and Yinhong Zhao. Empirical analysis of eip-1559: Transaction fees, waiting times, and consensus security. In *Proceedings of the 2022 ACM SIGSAC Conference on Computer and Communications Security*, CCS '22, page 2099–2113, New York, NY, USA, 2022. Association for Computing Machinery. doi:10.1145/3548606.3559341.

[18] Kaihua Qin, Liyi Zhou, and Arthur Gervais. Quantifying blockchain extractable value: How dark is the forest? In *2022 IEEE Symposium on Security and Privacy (SP)*, pages 198–214. IEEE, 2022.

[19] Daniël Reijsbergen, Shyam Sridhar, Barnabé Monnot, Stefanos Leonardos, Stratis Skoulakis, and Georgios Piliouras. Transaction fees on a honeymoon: Ethereum's eip-1559 one month later. In *2021 IEEE International Conference on Blockchain (Blockchain)*, pages 196–204. IEEE, 2021.

[20] Tim Roughgarden. Transaction fee mechanism design for the Ethereum blockchain: An economic analysis of EIP-1559. *arXiv preprint arXiv:2012.00854*, 2020.

[21] Tim Roughgarden. Transaction fee mechanism design. *ACM SIGecom Exchanges*, 19(1):52–55, 2021.

[22] Christof Ferreira Torres, Ramiro Camino, et al. Frontrunner jones and the raiders of the dark forest: An empirical study of frontrunning on the ethereum blockchain. In *30th USENIX Security Symposium (USENIX Security 21)*, pages 1343–1359, 2021.

A Omitted Proofs

Theorem 1. *In expectation, it is rational for miner X to deviate from the honest strategy, if*

$$p_x > \frac{\varepsilon}{\phi \cdot b^* + (1-\alpha)\varepsilon}.$$

Proof. We commence with the honest strategy and calculate the expected payout for consecutive turns of X as proposer by modeling the process as a Markov chain (cf. Figure 7a).

When it is miner X's first time as proposer in a consecutive turn, i.e., the previous block was not mined by X, we are in state X^h. In this state miner X proposes a block at target size s^* and receives tips at price ε. Thus, the payout for miner X in state X^h is $P[X^h] = s^* \cdot \varepsilon$. With probability p_x, X stays in state X^h for the next block and also proposes the next block. Else, with probability $1 - p_x$, we enter an absorbing state. We calculate the expected number of times X proposes consecutive blocks and, thereby, the expected payout for X.

By linearity of expectation, it follows that the expected payout for a sequence of consecutive turns by X as a proposer following the honest strategy, is given by

$$\mathbb{E}[R[X^h]] = p_x \cdot \mathbb{E}[R[X^h]] + P[X^h] \Rightarrow \mathbb{E}[R[X^h]] = \frac{s^* \cdot \varepsilon}{1 - p_x}, \tag{8}$$

as miner X is awarded $P[X^h]$ every time he proposes a block. Note that the expected reward of the honest strategy — $\mathbb{E}[R[H]]$ — corresponds to the expected payout of the Markov process starting in state X^h, i.e., $\mathbb{E}[R[H]] = \mathbb{E}[R[X^h]]$.

(a) honest strategy of miner X (b) attacking strategy of miner X

Figure 7: The honest strategy (cf. Figure 7a) and deviation from the honest strategy (cf. Figure 7b) modeled with discrete Markov chains. All states with a nonzero payout for miner X are highlighted in gray. We transition between states with every block. Note that for all remaining probabilities, the Markov process enters an absorbing state and X's consecutive turns as a proposer finish.

We now examine the reward for miner X if it chooses to carry out the attack, modelling the deviation from the honest strategy as a Markov chain (as shown in Figure 7b). The starting point for the attack, denoted as state S^a, is when miner X submits an empty block. Therefore, the payout in state S^a is 0. But with a probability of p_x, miner X is chosen as the proposer for the next block and enters

state X^a, where it submits a block at the target size. The payout in state X^a can be calculated by $P[X^a] = s^* (\phi \cdot b^* + (1-\alpha)\varepsilon)$.

If X is not selected to propose another block in a row, we enter an absorbing state and the attack stops. We make the approximation that whenever X's consecutive turns as proposer finish, honest miners will bring the base fee back to b^* before X gets to mine another block. It is possible that the honest miners do not bring the base fee back up to its target before X is selected to propose again, and X could continue the attack at a lower cost. However, assuming that the base fee had fully recovered simplifies the analysis and only makes our results stronger, as it reduces X's attack rewards. From state X^a we remain in state X^a for the next block with probability p_x, i.e., X is selected to propose another block, or enter the absorbing state with probability $1 - p_x$.

The expected payout for miner X in state S^a is given by $\mathbb{E}[R[S^a]] = p_x \cdot \mathbb{E}[R[X^a]] + P[S^a]$, where $\mathbb{E}[R[X^a]]$ is the expected payout starting from state X^a and we have $\mathbb{E}[R[X^a]] = p_x \cdot \mathbb{E}[R[X^a]] + P[X^a]$. It follows that the expected payout of the attack is

$$\mathbb{E}[R[A]] = \mathbb{E}[S^a] = \frac{p_x \cdot s^* (\phi \cdot b^* + (1-\alpha)\varepsilon)}{(1-p_x)}. \tag{9}$$

Note that the payout of the the attack is given by the expected payout starting from state S^a, as the attack commences in said state.

We conclude that it is rational for miner X to deviate from the honest strategy if $\mathbb{E}[R[A]] > \mathbb{E}[R[H]]$. It follow that X attacks when

$$p_x > \tfrac{\varepsilon}{\phi \cdot b^* + (1-\alpha)\varepsilon}.$$

□

Theorem 2. *In expectation, it is rational for a miner Y to deviate from the honest strategy and join X in keeping the base fee low, if*

$$p_y > \frac{\Delta((1-\alpha)\varepsilon + \phi \cdot b^*)}{(1-\alpha)\Delta \cdot \varepsilon + (1+\Delta)\phi \cdot b^* - \alpha \cdot \varepsilon}.$$

Proof. We commence with the honest strategy of miner Y which we model as a Markov chain in Figure 8a. Miner Y starts in state Y_X^h and is tasked with proposing a block after miner X at an artificially lowered base fee b, where $b = (1-\phi)b^* < b^*$. The payout for proposing a block at twice the target size s^* is given by

$$P[Y_X^h] = s^* (\phi \cdot b^* + (1-\alpha)\varepsilon)(1+\Delta). \tag{10}$$

From state Y_X^h, we move to state Y_Y^h, where Y proposes a target size block, with probability p_y. The payout for miner Y in state Y_Y^h is given as

$$P[Y_Y^h] = s^* \cdot \varepsilon, \tag{11}$$

and we remain in this state for a subsequent block with probability p_y.

In both state Y_X^h and state Y_Y^h, the probability of moving to state X^h is p_x. When we re-enter state X^h for the first time, X will propose an empty block to lower the base fee again. Then, with a probability p_x, we remain in state X for the next block, where miner X will propose target size blocks until its consecutive turn as a proposer is interrupted. Regardless, the payout for miner Y whenever we are in state X^h is zero. We move to state Y_Y^h with probability of p_y from state X^h and enter an absorbing state with probability $1 - p_y - p_x$ from all states.

Next, we calculate the expected payout of the honest strategy and start with the expected reward for miner Y starting from state Y_X^h $\mathbb{E}[R[Y_X^h]] = p_x \cdot \mathbb{E}[R[X^h]] + p_y \cdot \mathbb{E}[R[Y_Y^h]] + P[Y_X^h]$ where $\mathbb{E}[R[X^h]] = p_x \cdot \mathbb{E}[R[X^h]] + p_y \cdot \mathbb{E}[R[Y_X^h]]$, and $\mathbb{E}[R[Y_Y^h]] = p_x \cdot \mathbb{E}[R[X^h]] + p_y \cdot \mathbb{E}[R[Y_Y^h]] + P[Y_Y^h]$. By solving the system of linear equation, we find that the expected reward from the honest strategy is given by

$$\mathbb{E}[R[H]] = \mathbb{E}[R[Y_X^h]] = \tfrac{(1-p_x)(((1-p_y)(1+\Delta)\phi \cdot b^*) + \varepsilon(1+\Delta-\alpha(1+\Delta)(1-p_y)-\Delta p_y))s^*}{(1-p_x-p_y)}. \tag{12}$$

We now consider the deviation from the honest strategy, whereby miner Y also keeps the base fee artificially low, and model the strategy with a Markov chain in Figure 8b. State Y^a is the starting state of the deviating strategy in which Y proposes a block with target size s^* at an artificially lowered base fee $(1-\phi)b^*$. Thus, the state's payout is

$$P[Y^a] = s^* (\phi \cdot b^* + (1-\alpha)\varepsilon). \tag{13}$$

(a) honest strategy of miner Y (b) attacking strategy of miner Y

Figure 8: The honest strategy (cf. Figure 8a) and the deviation from the honest strategy, i.e., keep the base fee artificially low, (cf. Figure 8b) modeled with discrete Markov chains for miner Y. We transition between states with every block. All state with a nonzero payout for miner Y are highlighted in gray. Note that for all remaining probabilities, the Markov process enters an absorbing state.

For the next block, we stay in state Y^a with probability p_y, move to state X^a with probability p_x, or else move to an absorbing state, i.e., the consecutive turn of X and Y as proposers ends and the attack finishes. In state X^a, proposer X will also propose a target size block, but the payout for miner Y is zero as we assume no collaboration between the two. The transition probabilities from state X^a are identical to those from state Y^a: we move to state Y^a with probability p_y, stay in state X^a with probability p_x and enter an absorbing state with probability $1 - p_y - p_x$ for the next block. Thus, the expected reward for miner Y starting in state Y^a is given by $\mathbb{E}[R[Y^a]] = p_x \cdot \mathbb{E}[R[X^a]] + p_y \cdot \mathbb{E}[R[Y^a]] + P[Y^a]$, where $\mathbb{E}[R[X^a]]$ is the expected payout for miner Y starting from state X^a and we have $\mathbb{E}[R[X^a]] = p_x \cdot \mathbb{E}[R[X^a]] + p_y \cdot \mathbb{E}[R[Y^a]]$. We follow that the expected payout of the deviating strategy for miner Y is given by

$$\mathbb{E}[R[A]] = \mathbb{E}[R[Y^a]] = \frac{(1-p_x)s^*(\phi \cdot b^* + (1-\alpha)\varepsilon)}{(1-p_x-p_y)}. \tag{14}$$

We conclude it is rational for miner Y to deviate from the honest strategy when $\mathbb{E}[R[A]] - \mathbb{E}[R[H]] > 0$. It follows that Y attacks when

$$p_y > \frac{\Delta((1-\alpha)\varepsilon + \phi \cdot b^*)}{(1-\alpha)\Delta \cdot \varepsilon + (1+\Delta)\phi \cdot b^* - \alpha \cdot \varepsilon}.$$

□

Theorem 3. *In expectation, it is rational for a miner Y to deviate from the honest strategy and lower the base fee, if*

$$p_y > \frac{\varepsilon(1-p_x)}{(1-\alpha)\varepsilon + \phi \cdot b^*(1-p_x) + (1+\Delta)\varepsilon\alpha p_x - \Delta p_x(\varepsilon + \phi \cdot b^*)}.$$

Proof. We, again, model the honest strategy for miner Y as a Markov chain (cf. Figure 9a). The honest strategy starts in state Y^h, where miner Y proposes a block at target size s^* and receives a payout of

$$P[Y^h] = s^* \cdot \varepsilon. \tag{15}$$

The transition probability to state X^h is p_x and to state Y^h, i.e., Y proposes consecutive blocks, is p_y. Else, the consecutive turn of X and Y as miners stops and we enter an absorbing state. We note that the states X^h, Y_X^h and Y_Y^h correspond exactly to the eponymous states in Theorem 2 (cf. Figure 8a). Thus, the actions of the miners, the payout for miner Y and the transition probabilities are as previously described in the proof of Theorem 2.

The strategy's payout is the expected reward starting from state Y^h, which is

$$\mathbb{E}[R[Y^h]] = p_x \cdot \mathbb{E}[R[X^h]] + p_y \cdot \mathbb{E}[R[Y^h] + P[Y^h]. \tag{16}$$

As previously in Theorem 2, we have

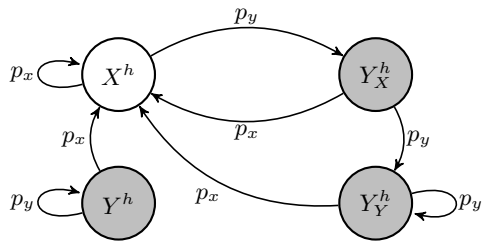
$$\mathbb{E}[R[Y_X^h]] = p_x \cdot \mathbb{E}[R[X^h]] + p_y \cdot \mathbb{E}[R[Y_Y^h]] + P[Y_X^h], \tag{17}$$

$$\mathbb{E}[R[X^h]] = p_x \cdot \mathbb{E}[R[X^h]] + p_y \cdot \mathbb{E}[R[Y_X^h]], \tag{18}$$

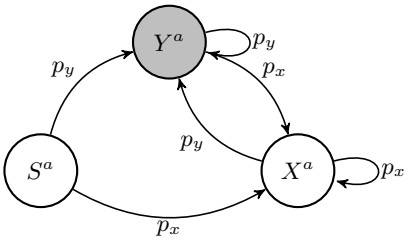
$$\mathbb{E}[R[Y_Y^h]] = p_x \cdot \mathbb{E}[R[X^h]] + p_y \cdot \mathbb{E}[R[Y_Y^h]] + P[Y_Y^h]. \tag{19}$$

Solving the system of linear equation, we conclude that the expected payout of the honest strategy is

$$\mathbb{E}[R[H]] = \mathbb{E}[R[Y^h]] = \frac{(((1+\Delta)b^*\phi p_x p_y) + \varepsilon(1 - p_x + ((1-\alpha)\Delta - \alpha)p_x p_y))s^*}{1-p_x-p_y}. \tag{20}$$



(a) honest strategy of miner Y



(b) attacking strategy of miner Y

Figure 9: The honest strategy (cf. Figure 9a) and the deviation from the honest strategy (cf. Figure 9b) modeled with discrete Markov chains. We transition between states with every block. All state with a nonzero payout for miner Y are highlighted in gray. Note that for all remaining probabilities, the Markov process enters an absorbing state and the consecutive turn of X and Y as proposers finishes.

We proceed with the attack strategy, which we model in Figure 9b. Miner Y starts in state S^a and mines an empty block and, therefore, receives no rewards. With probability p_y we move to state Y^a for the next block, i.e., Y proposes a target size (s^*) block, with probability p_x we move to state X^a, i.e., X proposes a target size (s^*) block, and with probability $1 - p_x - p_y$ we move to an absorbing state, i.e., the attack ends. Notice that the states X^a and Y^a are identical to those described in proof of Theorem 3. Thus, we the expected returns starting the respective states are as follows

$$\mathbb{E}[R[S^a]] = p_x \cdot \mathbb{E}[R[X^a]] + p_y \cdot \mathbb{E}[R[Y^a]], \tag{21}$$

$$\mathbb{E}[R[Y^a]] = p_x \cdot \mathbb{E}[R[X^a]] + p_y \cdot \mathbb{E}[R[Y^a]] + P[Y^a], \tag{22}$$

$$\mathbb{E}[R[X^a]] = p_x \cdot \mathbb{E}[R[X^a]] + p_y \cdot \mathbb{E}[R[Y^a]], \tag{23}$$

and find that the expected reward of the attack strategy is

$$\mathbb{E}[R[A]] = \mathbb{E}[R[S^a]] = \frac{(\phi \cdot b^* + (1-\alpha)\varepsilon)p_y s^*}{1 - p_x - p_y}. \tag{24}$$

To conclude, it is rational behavior for Y to deviate from the honest strategy, when $\mathbb{E}[R[A]] - \mathbb{E}[R[H]] > 0$, which holds when

$$p_y > \frac{\varepsilon(1-p_x)}{(1-\alpha)\varepsilon + \phi \cdot b^*(1-p_x) + (1+\Delta)\varepsilon\alpha p_x - \Delta p_x(\varepsilon + \phi \cdot b^*)}.$$

□

B Delay Incurred by the Mitigation of Section 6

Suppose we are in a steady state with a base fee b^*, when at after block height τ a fixed change in demand occurs for which the new steady state will be achieved with the new base fee $\beta \cdot b^*$. We denote by T the number of consecutively full blocks is takes to reach the new base fee $\beta \cdot b^*$.

After k consecutively full blocks, according to Eq. 5

$$\begin{aligned} s_{avg}[\tau + k] &= (1-q)2s^* + q \cdot s_{avg}[\tau + k - 1] \\ &= \left(2(1-q)(q^0 + q^1 + \ldots + q^{k-1}) + q^k\right) s^* \\ &= \left(2(1-q)\left(\frac{1-q^k}{1-q}\right) + q^k\right) s^* \\ &= (2 - q^k)s^*. \end{aligned}$$

Plugging the above into Eq. 6 yields

$$\begin{aligned} b[\tau + k] &= b_{avg}[\tau + k - 1] \cdot \left(1 + \phi \cdot \frac{(2-q^k)s^* - s^*}{s^*}\right) = b_{avg}[\tau + k - 1] \cdot \left(1 + \phi(1 - q^k)\right) \\ &= b[\tau] \cdot \left(1 + \phi(1-q^1)\right)\left(1 + \phi(1-q^2)\right) \cdots \left(1 + \phi(1-q^k)\right) \\ &= b^* \cdot \prod_{i=1}^{k} (1 + \phi(1 - q^i)). \end{aligned}$$

Therefore, T is the smallest integer that satisfies

$$b[\tau + T] = b^* \cdot \prod_{i=k}^{T}(1 + \phi(1 - q^k)) \geq \beta \cdot b^* \quad \iff \quad \prod_{k=1}^{T}(1 + \phi(1 - q^k)) \geq \beta.$$

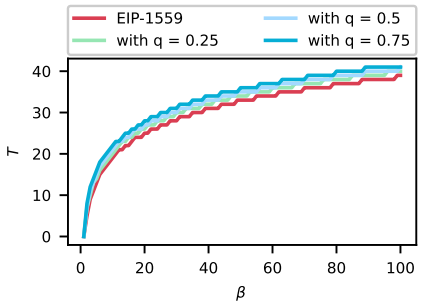


Figure 10: The number of consecutive full blocks (T) required to increase the base fee by factor β for EIP-1559 and the proposed mitigation for $q \in \{1/4, 1/2, 3/4\}$.

Figure 10 plots $T(\beta)$ for both EIP-1559 and our mitigation. We set $\phi = 1/8$ (current Ethereum) and use $q \in \{1/4, 1/2, 3/4\}$. All plots follow a logarithmic trend and that the response times to dramatic changes in demand are only mildly affected by the proposed mitigation (even for β factors as large as 100).